DATE:_____

Derivatives of Logarithmic Functions

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$y = \ln x \rightarrow e^y = x$

$e^y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$= \frac{1}{x}$

Example 1:

Given $f(x) = 2\ln x$, find $f'(x)$.

$f'(x) = 2 \cdot \frac{1}{x}$

$\boxed{f'(x) = \frac{2}{x}}$

Example 2:

Given $g(x) = \ln(2x)$ find $g'(x)$.

$g'(x) = 2 \cdot \frac{1}{2x}$

$\boxed{g'(x) = \frac{1}{x}}$

OR $g(x) = \ln 2 + \ln x$

$g'(x) = \frac{1}{x}$

$\ln ab = \ln a + \ln b$

Example 3:

Given $h(x) = \ln\left(\frac{x}{2x+3}\right)$, find $h'(x)$.

quotient :

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$= \ln x - \ln(2x+3)$

$h'(x) = \frac{1}{x} - 2 \cdot \frac{1}{2x+3}$

$\frac{(2x+3)}{(2x+3)}\frac{1}{x} - \frac{2}{x+3}\frac{(x)}{(x)}$

$= \frac{2x+3-2x}{x(x+3)}$

$\boxed{h'(x) = \frac{3}{x(x+3)}}$

Example 4:

product + chain

Find $f'(x)$ where $f(x) = \ln((x+3)^2(4x+1))$

$\ln ab = \ln a + \ln b$

$\ln a^b = b \ln a$

$= \ln(x+3)^2 + \ln(4x+1)$

$= 2\ln(x+3) + \ln(4x+1)$

$f'(x) = 2 \cdot \frac{1}{x+3} + 4 \cdot \frac{1}{4x+1}$

$\frac{2}{x+3}\frac{(4x+1)}{(4x+1)} + \frac{4}{4x+1}\frac{(x+3)}{(x+3)}$

OR $= \frac{8x+2+4x+12}{(4x+1)(x+3)}$

$$\boxed{f'(x) = \frac{12x+14}{(4x+1)(x+3)}}$$

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}}$$

remember

$\log_a b = \frac{\log_c b}{\log_c a}$

$\log_a b = \frac{\ln b}{\ln a}$

$\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right)$

$= \frac{d}{dx}\left(\frac{1}{\ln a} \cdot \ln x\right)$

$= \frac{1}{\ln a} \cdot \frac{1}{x}$

Example 1:

Find $\frac{dy}{dx}$, where $y = log_8 x$.

$\frac{dy}{dx} = \frac{1}{\ln 8} \cdot \frac{1}{x}$

$$\boxed{\frac{dy}{dx} = \frac{1}{x \ln 8}}$$

Example 2:

Find y' for $y = log_5(3x+2)$.

$y' = \frac{1}{\ln 5} \cdot 3 \cdot \frac{1}{3x+2}$

$$\boxed{y' = \frac{3}{\ln 5(3x+2)}}$$