DATE: _____

Trapezoid Rule

Recall Riemann Sums….

$\Delta x = \frac{1}{2}$ $\Delta x = \frac{1}{2}$ $\Delta x = \frac{1}{2}$ $\Delta x = \frac{1}{2}$

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	4	5	8	13
		R	R	R	R

A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, what is the right sum approximation of $\int_0^2 f(x)\,dx$?

$a \quad b$

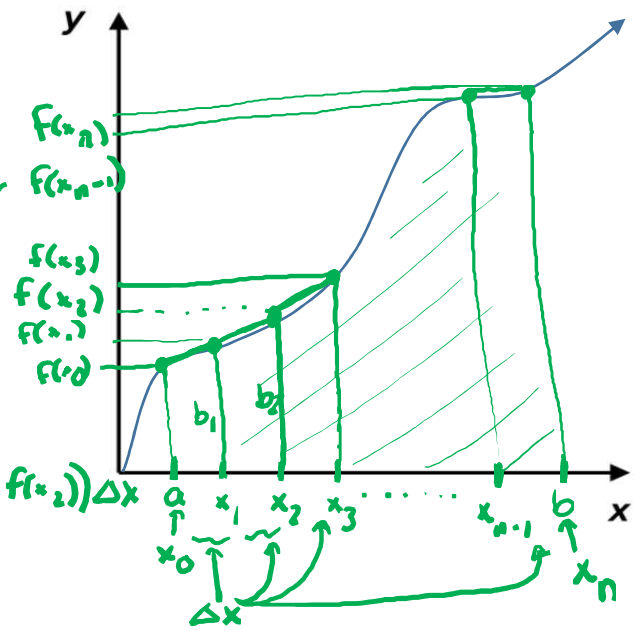
$\Delta x = \frac{b-a}{n} \rightarrow \Delta x = \frac{2-0}{4} = \frac{1}{2}$

$$\int_0^2 f(x)\,dx \approx \frac{1}{2}(13) + \frac{1}{2}(8) + \frac{1}{2}(5) + \frac{1}{2}(4)$$
$$\approx \frac{1}{2}(13+8+5+4)$$
$$\approx \frac{1}{2}(30)$$
$$\approx \boxed{15}$$

Trapezoid Sum (Trapezoid Rule)

Area of a Trapezoid: $\frac{1}{2}(b_1 + b_2)h$

$\int_a^b f(x)\,dx \rightarrow$ trapezoid approximation of the integral.



Trapezoid Rule

$$= \frac{1}{2}(f(x_0) + f(x_1))\Delta x + \frac{1}{2}(f(x_1) + f(x_2))\Delta x$$
$$+ \frac{1}{2}(f(x_2) + f(x_3))\Delta x + \cdots$$
$$+ \frac{1}{2}(f(x_{n-1}) + f(x_n))\Delta x$$

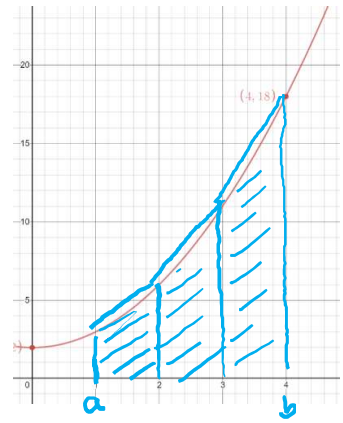
$$= \frac{1}{2}\Delta x\left(f(x_0) + f(x_1) + f(x_1) + f(x_2) + f(x_2) + f(x_3) + \cdots + f(x_{n-1}) + f(x_n)\right)$$

$$= \frac{1}{2}\left(\frac{b-a}{n}\right)\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right)$$

Example 1:

$n \to \Delta x = \frac{b-a}{n}$

Use a trapezoidal sum with 3 equal subintervals to estimate the area of the region bounded by $y = x^2 + 2$ and x-axis between $x = 1$ (a) and $x = 4$ (b).



x	1	2	3	4
$f(x)$	3	6	11	18

$\Delta x = \frac{4-1}{3} = 1$

$$\int_1^4 (x^2+2)\,dx \approx \frac{1}{2}\left(\frac{4-1}{3}\right)\left(3 + 2(6) + 2(11) + 18\right)$$
$$\approx \frac{1}{2}(1)(3 + 12 + 22 + 18)$$
$$\approx \frac{1}{2}(55)$$
$$\approx \frac{55}{2}$$

Example 2:

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	4	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, what is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

$n = 4$ $\quad a \quad b$

$$\int_0^2 f(x)\,dx \approx \frac{1}{2}\left(\frac{2-0}{4}\right)\left(3 + 2(4) + 2(5) + 2(8) + 13\right)$$
$$\approx \frac{1}{2}\left(\frac{1}{2}\right)(3 + 8 + 10 + 16 + 13)$$
$$\approx \frac{1}{4}(50)$$
$$\approx \boxed{\frac{25}{2}}$$

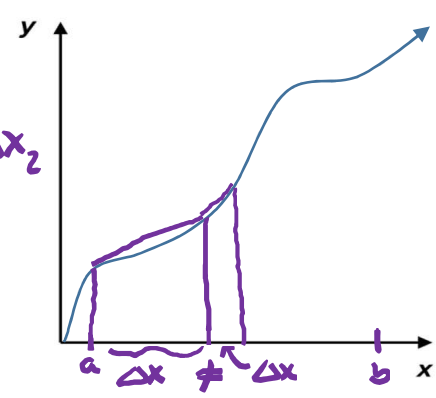
What if the subintervals are not equal?

Trapezoid Sum (Unequal Subintervals)

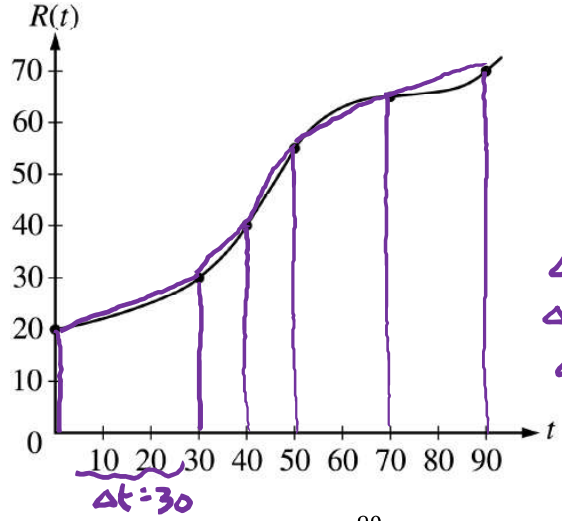
$\int_a^b f(x)\,dx \to$ trapezoid approximation

$$\int_a^b f(x)\,dx \approx \frac{1}{2}\left(f(x_0) + f(x_1)\right)\Delta x_1 + \frac{1}{2}\left(f(x_1) + f(x_2)\right)\Delta x_2 + \dots$$

need to calculate area of each trapezoid separately.



Example 1:



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

Δt = 30, Δt = 10, Δt = 10, Δt = 20, Δt = 20

cannot use trap rule!

Approximate the value of $\int_0^{90} f(x)\,dx$ using a trapezoidal sum with the five subintervals indicated by the data in the table.

$n = 5$

$$\int_0^{90} f(x)\,dx \approx \tfrac{1}{2}(20+30)30 + \tfrac{1}{2}(30+40)10 + \tfrac{1}{2}(40+55)(10) + \tfrac{1}{2}(55+65)(20) + \tfrac{1}{2}(65+70)(20)$$

$$\approx \tfrac{1}{2}\left(50(30) + 70(10) + 95(10) + 120(20) + 135(20)\right)$$

$$\approx \tfrac{1}{2}(8250)$$

$$\approx 4125 \text{ gallons}$$

Example 2:

Δx = 1, Δx = 3, Δx = 2, Δx = 4

x	0	1	4	6	10
$f(x)$	3	5	2	-1	1

A table of values for a continuous function f is shown above. If four subintervals of $[0, 10]$ are used, what is the trapezoidal approximation of $\int_0^{10} f(x)\,dx$?

$$\int_0^{10} f(x)\,dx \approx \tfrac{1}{2}(3+5)(1) + \tfrac{1}{2}(5+2)(3) + \tfrac{1}{2}(2+-1)(2) + \tfrac{1}{2}(-1+1)(4)$$

$$\approx \tfrac{1}{2}\left(8(1) + 7(3) + 1(2) + 0(4)\right)$$

$$\approx \tfrac{1}{2}(8+21+2)$$

$$\approx \tfrac{1}{2}(31)$$

$$\approx \boxed{\tfrac{31}{2}}$$