

2023



AP[®] Calculus AB

Free-Response Questions

Answer QUESTION 1 parts (a) and (b) on this page.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Response for question 1(a)

$$\int_{60}^{135} f(t) dt \approx 15(0.05) + 30(0.1) + 30(0.15) \text{ gallons}$$

1 pt: right sum
1 pt: answer

$\int_{60}^{135} f(t) dt$ is the amount of gasoline that flowed in gallons from 60 to 135 seconds.

1 pt: sentence w/ units

Response for question 1(b)

$f(t)$ is diff'able on $(60, 120)$ b/c given
 $f(t)$ is cont on $[60, 120]$ b/c $f(t)$ is diff'able.

$$\begin{aligned} f'(c) &= \frac{f(120) - f(60)}{120 - 60} \\ &= \frac{0.1 - 0.1}{120 - 60} \\ &= 0 \end{aligned}$$

1 pt: $f(120) - f(60) = 0$

\therefore , there must exist a value of c on $(60, 120)$ such that $f'(c) = 0$

1 pt: answer w/ justification

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$$\begin{aligned} \text{avg rate of flow} &= \frac{1}{150-0} \int_0^{150} g(t) dt \\ &= 0.096 \end{aligned}$$

1 pt: avg value formula

1 pt: answer

Response for question 1(d)

$$g'(140) = -0.005 \text{ gallons/sec}^2$$

The rate of flow of gasoline, in gallons/sec²,

@ $t = 140$ seconds is decreasing.

1 pt: $g'(140)$

1 pt: sentence w/ units

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

Stephen change direction $\rightarrow v(t)$ change sign

1pt: considers sign of $v(t)$

Stephen changes direction @ $t = 56$

b/c $v(t)$ changes sign @ $t = 56$

1pt: answer w/ reason

Response for question 2(b)

$$v'(60) = a(60) = -0.036 \text{ m/sec}^2$$

1pt: $a(60)$ w/ setup

$$v(60) = -0.160$$

1pt: units

Stephen is speeding up @ $t = 60$

b/c $v(60) < 0$ and $a(60) < 0$

1pt: speed up w/ reason

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$x(20) = \int_0^{20} v(t) dt$$

$$x(80) = \int_0^{80} v(t) dt$$

$$x(80) - x(20) = \int_{20}^{80} v(t) dt$$

$$= 23.384$$

1pt: integral

1pt: answer

$$\int_{20}^{80} v(t) dt = x(t) \Big|_{20}^{80} \dots \text{☺}$$
$$= x(80) - x(20)$$

Response for question 2(d)

$$\text{total distance} = \int_0^{90} |v(t)| dt$$

$$= 62.164$$

1pt: integral

1pt: answer