2023

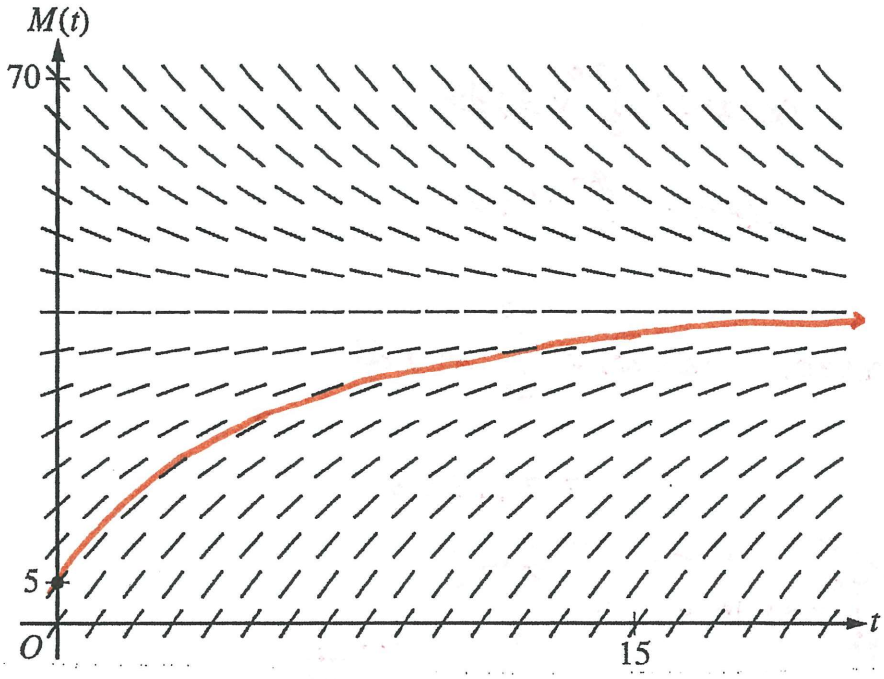
AP

AP® Calculus AB

Free-Response Questions

Answer QUESTION 3 parts (a) and (b) on this page.

Response for question 3(a)



1pt: solution curve (approaches, but doesn't touch or cross asymptote)

Response for question 3(b)

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{35}{4}(x - 0)$

$y = \frac{35}{4}(x) + 5$

$\frac{dM}{dt} = \frac{1}{4}(40 - 5)$

$= \frac{35}{4}$

1pt: $\left.\frac{dM}{dt}\right|_{t=0}$

1pt: approx.

$M(2) \approx \frac{35}{4}(2) + 5$ ← ok to stop here

$\approx \frac{35}{2} + 5$

$\approx \frac{45}{2}$

Answer QUESTION 3 parts (c) and (d) on this page.

Response for question 3(c)

$$\frac{d^2M}{dt^2} = \frac{1}{4} \cdot -\frac{dM}{dt}$$

$$= -\frac{1}{4} \cdot \frac{1}{4}(40-M)$$ ← ok to stop here

$$= -\frac{1}{16}(40-M)$$

1pt: $\frac{d^2M}{dt^2}$

Approx from part (b) is overestimate b/c $\frac{d^2M}{dt^2} < 0$ on $(0,2)$

1pt: over w/ reason

Response for question 3(d)

$$\frac{dM}{dt} = \frac{1}{4}(40-M)$$

$$dM = \frac{1}{4}(40-M)\,dt$$

$$\int \frac{1}{40-M}\,dM = \int \frac{1}{4}\,dt$$

u-sub
$u = 40-M$
$du = -dM$
$-du = dM$

$$-\int \frac{1}{u}\,du = \frac{1}{4}t + C$$

$$-\ln|40-M| = \frac{1}{4}t + C \rightarrow -\ln|40-M| = \frac{1}{4}t - \ln 35$$

$$-\ln|40-5| = \frac{1}{4}(0) + C$$

$$-\ln 35 = C$$

$$\ln|40-M| = -\frac{1}{4}t + \ln 35$$

$$|40-M| = e^{-\frac{1}{4}t + \ln 35}$$

$40 - M > 0$ so, keep +

$$40 - M = e^{-\frac{1}{4}t + \ln 35}$$

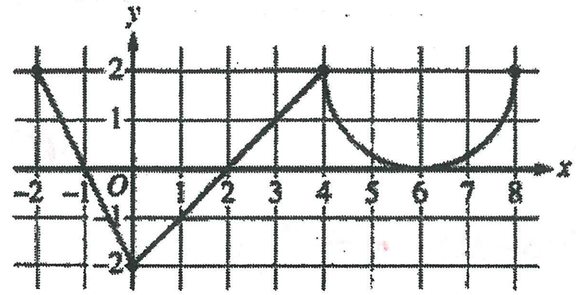
$$-M = e^{-\frac{1}{4}t + \ln 35} - 40$$

$$M = -e^{-\frac{1}{4}t + \ln 35} + 40$$

OR $M = -35e^{-\frac{1}{4}t} + 40$

1pt: separate
1pt: antiderivative
1pt: ~~constant~~ initial condition
1pt: solves for M.

Answer QUESTION 4 parts (a) and (b) on this page.



Graph of f'

Response for question 4(a)

f has neither a rel max nor rel min @ $x=6$

b/c f' does not change signs @ $x=6$.

1pt: answer w/ reason.

Response for question 4(b)

f is concave down on $(-2,0) \cup (4,6)$

b/c $f''<0$ on $(-2,0) \cup (4,6)$

1pt: intervals

1pt: reason

☺ ... or slope of $f'<0$

or f' dec

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x\to 2} \frac{6f(x)-3x}{x^2-5x+6} \quad \begin{matrix} \to 6f(2)-6 \to 0 \\ \to 4-10+6 \to 0 \end{matrix}$$

1pt: limits of numerator + denominator

$\therefore$, by L'Hôpital's rule,

1pt: uses L'Hôpital

$$\lim_{x\to 2} \frac{6f(x)-3x}{x^2-5x+6} = \lim_{x\to 2} \frac{6f'(x)-3}{2x-5}$$

$$= \frac{6f'(2)-3}{2(2)-5}$$

$$= \frac{6(0)-3}{2(2)-5}$$

← ok to stop here

1pt: answer

$$= 3$$

Response for question 4(d)

$$f(x) = 1 + \int_2^x f'(t)\,dt$$

1pt: $f'=0$
1pt: justification
1pt: answer

$f'(x) = 0$ @ $x = -1, 2, 6$

$$f(-2) = 1 + \int_2^{-2} f'(x)\,dx$$
$$= 1 - \int_{-2}^{2} f'(x)\,dx$$
$$= 1 - \left(-\tfrac{1}{2}(2)(2)\right)$$
$$= 3$$

$$f(-1) = 1 + \int_2^{-1} f'(x)\,dx$$
$$= 1 - \int_{-1}^{2} f'(x)\,dx$$
$$= 1 - \left(-\tfrac{1}{2}(3)(2)\right)$$
$$= 4$$

$f(2) = 1$

$$f(6) = 1 + \int_2^6 f'(x)\,dx$$
$$= 1 + \tfrac{1}{2}(2)(2) + 4(2) - \tfrac{1}{4}\pi(2)^2$$
$$> 1$$

$$f(8) = 1 + \int_2^8 f'(x)\,dx$$
$$= 1 + \tfrac{1}{2}(2)(2) + 4(2) - \tfrac{1}{2}\pi(2)^2$$
$$> 1$$

Abs min value is 1.

Answer QUESTION 5 parts (a) and (b) on this page.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	−8	3	6
$g(x)$	1	2	−3	0
$g'(x)$	5	4	2	8

Response for question 5(a)

$h(x) = f(g(x))$

$h'(x) = g'(x) \cdot f'(g(x))$ — 1pt: chain rule

$h'(7) = g'(7) \cdot f'(g(7))$

$= 8 \cdot f'(0)$

$= 8 \cdot \frac{3}{2}$ ← ok to stop here — 1pt: answer

$= 12$

Response for question 5(b)

$k'(x) = (f(x))^2 \cdot g(x)$ — 1pt: product OR chain

$k''(x) = g(x) \cdot f'(x) \cdot 2f(x) + (f(x))^2 \cdot g'(x)$

$k''(4) = g(4) \cdot f'(4) \cdot 2f(4) + (f(4))^2 \cdot g'(4)$ — 1pt: $k''(4)$

$= -3 \cdot 3 \cdot 2 \cdot 4 + (4)^2 \cdot 2$ ← ok to stop here

$= -72 + 32$

< 0

k is concave down @ $x = 4$ b/c $k''(4) < 0$ — 1pt: answer w/ reason

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$m(x) = 5x^3 + \int_0^x f'(t)\,dt$

$m(2) = 5(2)^3 + \int_0^2 f'(t)\,dt$

$= 5(2)^3 + f(t)\Big|_0^2$

$= 5(2)^3 + f(2) - f(0)$

$= 5(2)^3 + 7 - 10$ ← ok to stop here

$= 37$

1pt: answer w/ supporting work

Response for question 5(d)

$m'(x) = 15x^2 + f'(x)$

$m'(2) = 15(2)^2 + f'(2)$

$= 15(2)^2 + -8$ ← ok to stop here

> 0

m is inc @ $t=2$ b/c $m'(2) > 0$

1pt: considers $m'(x)$

1pt: $m'(2)$ w/ work

1pt: answer w/ reason.

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$6xy = 2 + y^3$

$6(y \cdot 1 + x\frac{dy}{dx}) = 3y^2\frac{dy}{dx}$

1pt: implicit

$6y + 6x\frac{dy}{dx} = 3y^2\frac{dy}{dx}$

$6y = 3y^2\frac{dy}{dx} - 6x\frac{dy}{dx}$

$6y = \frac{dy}{dx}(3y^2 - 6x)$

$\frac{6y}{3y^2 - 6x} = \frac{dy}{dx}$

1pt: verification

$\frac{2y}{y^2 - 2x} = \frac{dy}{dx}$

Response for question 6(b)

$\frac{dy}{dx} = 0$

1pt: $2y = 0$

$\frac{2y}{y^2 - 2x} = 0$

$2y = 0$ $\quad 6xy = 2 + y^3$

$y = 0$ $\quad 6x(0) \neq 2 + 0^3$

$0 \neq 2$

1pt: answer w/ reason

$\frac{dy}{dx} = 0$ only @ $y = 0$ but no pt on curve exists where $y = 0$

$\therefore$, no pt. when tangent line is horizontal.

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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$\frac{dy}{dx} = \frac{1}{0}$

$\frac{y}{y^2-2x} = \frac{1}{0}$

$y^2 - 2x = 0$

$y^2 = 2x$

$\frac{1}{2}y^2 = x$

$\frac{1}{2}(1)^2 = x$

$\frac{1}{2} = x$

$6xy = 2 + y^3$

$6(\frac{1}{2}y^2)\cdot y = 2 + y^3$

$3y^3 = 2 + y^3$

$2y^3 = 2$

$y^3 = 1$

$y = 1$

∴, tangent line vertical @ $(\frac{1}{2}, 1)$

1pt: $y^2 - 2x = 0$

1pt: subs in $x = \frac{1}{2}y^2$ into $6xy = 2 + y^3$

1pt: answer

Response for question 6(d)

$6xy = 2 + y^3$

$6(y \cdot \frac{dx}{dt} + x\frac{dy}{dt}) = 3y^2\frac{dy}{dt}$

$6(-2 \cdot \frac{2}{3} + \frac{1}{2}\frac{dy}{dt}) = 3(-2)^2\frac{dy}{dt}$

$-8 + 3\frac{dy}{dt} = 12\frac{dy}{dt}$

$-8 = 9\frac{dy}{dt}$

$\frac{dy}{dt} = -\frac{8}{9}$

1pt: implicit w/ respect to t.

1pt: answer.