

2023



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# AP<sup>®</sup> Calculus BC

## Free-Response Questions

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Answer QUESTION 1 parts (a) and (b) on this page.

$t$ (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Response for question 1(a)

$$\int_{60}^{135} f(t) dt \approx 15(0.05) + 30(0.1) + 30(0.15) \text{ gallons}$$

1 pt: right sum  
1 pt: answer

$\int_{60}^{135} f(t) dt$  is the amount of gasoline that flowed in gallons from 60 to 135 seconds.

1 pt: sentence w/  
units

Response for question 1(b)

$f(t)$  is diff'able on  $(60, 120)$  b/c given

$f(t)$  is cont on  $[60, 120]$  b/c  $f(t)$  is diff'able

$$\begin{aligned} f'(c) &= \frac{f(120) - f(60)}{120 - 60} \\ &= \frac{0.1 - 0.1}{120 - 60} \\ &= 0 \end{aligned}$$

1 pt:  $f(120) - f(60) = 0$

$\therefore$ , there must exist a value of  $c$  on  $(60, 120)$  such that  $f'(c) = 0$

1 pt: answer w/  
justification

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$$\begin{aligned} \text{avg rate} &= \frac{1}{150-0} \int_0^{150} g(t) dt \\ \text{of flow} &= 0.096 \end{aligned}$$

1pt: avg value formula

1pt: answer

Response for question 1(d)

$$g'(140) = -0.005 \text{ gallons/sec}^2$$

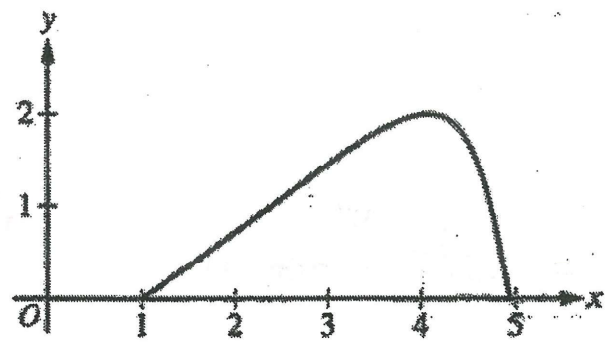
1pt:  $g'(140)$

The rate of flow of gasoline, in gallons/sec<sup>2</sup>,

@  $t = 140$  seconds is decreasing.

1pt: sentence by units

Answer QUESTION 2 parts (a) and (b) on this page.



Response for question 2(a)

$$a(t) = \langle x''(t), y''(t) \rangle$$

$$a(1) = \langle x''(1), y''(1) \rangle$$

$$= \langle -1.444, -6.683 \rangle$$

1 pt:  $x''(1)$  w/ set up  
 1 pt:  $y''(1)$  w/ set up

Response for question 2(b)

speed = 1.5

$$\sqrt{(x'(t))^2 + (y'(t))^2} = 1.5$$

$$t = 1.254$$

1 pt: equation  
 1 pt: answer



Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$\text{slope of tangent line} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\text{slope @ } t=1 = \frac{y'(1)}{x'(1)} = \frac{2 \cos 1}{e^{\cos 1}}$$

ok to stop here

1pt: slope w/ supporting work

$$= 0.630$$

$$x(1) = x(0) + \int_0^1 x'(t) dt$$

$$= 1 + \int_0^1 x'(t) dt$$

$$= 3.342$$

1pt: integral

1pt: x(1)

Response for question 2(d)

$$\text{total distance} = \int_0^{\pi} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$= 6.635$$

1pt: integral

1pt: answer