2023

AP

AP® Calculus BC

Free-Response Questions

Answer QUESTION 1 parts (a) and (b) on this page.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Response for question 1(a)

$\int_{60}^{135} f(t)\,dt \approx 15(0.05) + 30(0.1) + 30(0.15)$ gallons

1 pt: right sum
1 pt: answer

$\int_{60}^{135} f(t)\,dt$ is the amount of gasoline that flowed in gallons from 60 to 135 seconds.

1 pt: sentence w/ units

Response for question 1(b)

$f(t)$ is diff'able on $(60,120)$ b/c given

$f(t)$ is cont on $[60,120]$ b/c $f(t)$ is diff'able

$$f'(c) = \frac{f(120)-f(60)}{120-0}$$

$$= \frac{0.1-0.1}{120-0}$$

$$= 0$$

1 pt: $f(120)-f(60)=0$

$\therefore$, there must exist a value of c on $(60,120)$ such that $f'(c)=0$

1 pt: answer w/ justification

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

avg rate of flow $= \frac{1}{150-0}\int_0^{150} g(t)\,dt$

1pt: avg value formula

$= 0.096$

1pt: answer

Response for question 1(d)

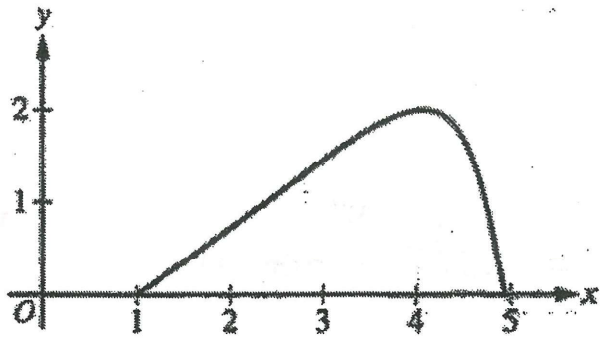
$g'(140) = -0.005$ gallons/sec^2

1pt: $g'(140)$

The rate of flow of gasoline, in gallons/sec^2, @ $t = 140$ seconds is decreasing.

1pt: sentence w/ units

Answer QUESTION 2 parts (a) and (b) on this page.



Response for question 2(a)

$a(t) = \langle x''(t), y''(t) \rangle$

$a(1) = \langle x''(1), y''(1) \rangle$

$= \langle -1.444, -.6683 \rangle$

1pt: $x''(1)$ w/ set up

1pt: $y''(1)$ w/ set up

Response for question 2(b)

speed = 1.5

$\sqrt{(x'(t))^2 + (y'(t))^2} = 1.5$

$t = 1.254$

1pt: equation

1pt: answer

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

slope of tangent line $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

slope @ $t=1$ $= \frac{y'(1)}{x'(1)} = \frac{2\cos 1}{e^{\cos 1}}$ ← ok to stop here

1pt: slope w/ supporting work

$= 0.630$

$x(1) = x(0) + \int_0^1 x'(t)\,dt$

$= 1 + \int_0^1 x'(t)\,dt$

$= 3.342$

1pt: integral

1pt: $x(1)$

Response for question 2(d)

total distance $= \int_0^{\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= 6.635$

1pt: integral

1pt: answer